Bachelor of Arts (B.A.) (Part—I) Semester—I Examination

STATISTICS

(Probability Theory)

Optional Paper—1

Time : Three Hours] [Maximum Marks : 50

1\. (A) Discuss the following three approaches to the definition of probability :

(i) Classical

(ii) Richard Von-Mises

(iii) Axiomatic.

Hence give classical, empirical and axiomatic definitions of probability stating relative merits and demerits of these definitions. 10

OR

(E) Define :

(i) Mutually exclusive events

(ii) Equally likely events

(iii) Exhaustive events

giving an example of each. If A and B are any two events then show that :

$P(A \cap \overline{B}) = P(A) - P(A \cap B)$.

If A is a subset of B, then show that $P(\overline{A} \cap B) = P(B) - P(A)$. Hence show that $P(A) \le P(B)$.

(F) Two fair dice are thrown. Let A denote the event that the numbers on the two dice differ by more than 2. Let B denote the event that the product of the two numbers is even. Find :

(i) P(A) & P(B)

(ii) $P(A \cap \overline{B})$

(iii) $P(A \cup B)$

(iv) $P(\overline{A} \cap \overline{B})$. 5+5

2\. (A) Define partition of a sample space. State and prove Baye's theorem. I travel to work by route A or route B. The probability that I choose route A is $\frac{1}{4}$. The probability that I am late for work if I go via route A is 2/5 and the corresponding probability if I go via route B is 1/3. Find the probability that :

(i) I am late for work on a day.

(ii) If I am late for work, I went via route B. 10

OR

(E) Define pair-wise and mutual independence of n events $A_1, A_2, \ldots, A_n$. If A, B and C are the events in the sample space such that these are pair-wise independent and A is independent of $B \cup C$. Then show that, A, B and C are mutually independent.

(F) Define conditional probability $P(A|B)$. State and prove multiplicative law for n events $A_1, A_2, \ldots, A_n$. If from a shipment of 20 television tubes of which 5 are defective tubes, we choose 2 television tubes one by one in succession without replacement. What is the probability that both will be defective tubes ? 5+5

3. (A) Define a random variable and a discrete random variable giving one example of each. Also define the probability mass function and cumulative distribution function of a random variable. A box contains 3 defective and 3 non-defective bolts. Suppose 3 bolts are picked at random from the box. Let X denote the number of defective bolts chosen in the sample. Find the pmf and cdf of X. Draw the graphs of pmf and cdf. 10

OR

(E) State and prove the properties of cumulative distribution function of a random variable. Let X be a r.v. with the following pdf f(x),

$$
\begin{aligned}
f(x) &= cx(2-x) \quad ; 0 \le x \le 2 \\
&= 0 \quad ; \text{Otherwise}
\end{aligned}
$$

Find :

(i) The value of c

(ii) cdf of X

(iii) $P[1/4 \le X \le 3/4]$

(iv) $P[X > 3/2]$

(v) $P[X \le 5/4]$. 10

4. (A) Define the three measures of location of a probability distribution. Explain how these are calculated for a discrete rv and for a continuous rv. Also, compare these measures. Let X be a r.v. with the following pmf :

x	0	1	2	3	4
p(x)	1/8	3/8	1/8	2/8	1/8

Find the mean, mode and median of X. 10

OR

(E) Define the r^{th} raw moment about origin and r^{th} central moment of the probability distribution of a r.v. Let X be a r.v. with the pdf f(x) given by, f(x) = 2x for $0 \le x \le 1$ and is zero otherwise. Find the first three raw moments of X about origin. Hence obtain μ_2, μ_3 and β_1. Comment upon the skewness of the probability distribution of X. 10

5. Solve any **ten** questions out of the following :

(A) 8 students are randomly selected to occupy 8 chairs around a circular table. What is the probability that two named students will be next to each other ?

(B) Define a discrete sample space giving an example.

(C) Can two mutually exclusive events be independent ? Justify the answer.

(D) If A and B are independent events then P(A|B) = ____ and P(B|A) = _____. (Fill in the blanks)

(E) From a pack of 52 cards, if a heart card is picked at random then what is the probability that it is a picture card ?

(F) A fair coin is tossed 2 times; Find the pmf of number of tails observed in 2 tosses of coin.

(G) Show that $P(A|B) + P(\overline{A}|B) = 1$.

(H) The cdf of a r.v. X is given by :

$$F(x) = \begin{cases} 0 & ; x < 0 \\ \dfrac{x^3}{27} & ; 0 \le x \le 3 \\ 1 & ; x > 3 \end{cases}$$

Find its pdf.

(I) Show that $V(aX) = a^2V(X)$, where a is constant and X is a r.v.

(J) Define the mgf of a r.v.

(K) If P(s) is the probability generating function of a r.v. X, then find the pgf of $\dfrac{X-a}{b}$.

(L) If the first two raw moments about the value 2 of the probability distribution of a r.v. X are 3 and 25 respectively, then find its mean and variance. 1×10=10